Experimental Investigation of Film Cooling Effectiveness for Slots of Various Exit Geometries

M. E. Taslim*

Northeastern University, Boston, Massachusetts 02115
and
S. D. Spring† and B. P. Mehlman‡
GE Aircraft Engines, Lynn, Massachusetts 01910

Nomenclature

H = mainstream plenum height

 $M = \text{blowing ratio}, (\rho U)_c/(\rho U)_h$

 \dot{m} = air mass flow rate

 $Re_s = \text{Slot Reynolds number, } (\rho Us/\mu)$

s = injection slot height

T = temperature

t = injection slot lip thickness

U = velocity

w = injection slot width

x =distance downstream from slot

 α = injection angle

 η_f = film effectiveness, $(T_h - T_{aw})/(T_h - T_c)$

v' = kinematic viscosity

 ρ = density

Subscripts

aw = adiabatic wall

c = property value at injection slot (cold) conditions

f = film

h = property value at mainstream air (hot) conditions

Introduction

N today's advanced jet engines, the limits of operating temperatures are continually pushed higher in order to meet present and future performance requirements. In fact, typical engine operating temperatures are much greater than the allowable turbine airfoil metal temperatures. This necessitates the requirement to provide thermal protection to the

airfoils. A common method of providing thermal protection to an airfoil trailing edge (often the life limiting area) is by injecting a film of cooling air through slots located on the airfoil pressure side near the trailing edge, thereby providing a cooling buffer between the hot mainstream gas and the airfoil surface (see Fig. 1). To determine the extent of thermal protection, it is crucial to predict the film effectiveness (η_f) provided by the cooling film.

Much research has been devoted to the topic of film cooling, and there exists a considerable amount of experimental data for film cooling with different slot geometries and various primary and secondary flow fields. A very comprehensive survey of film cooling was performed and compiled by Goldstein¹ which includes data for discrete holes as well as slots. It has been well established that the slot coolant to mainstream flow blowing ratio $(M = (\rho U)_c/(\rho U)_b)$ and nor-

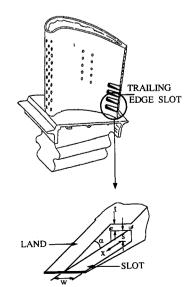


Fig. 1 Typical turbine airfoil with trailing edge slots.

Presented as Paper 90-2266 at the AIAA/SAE/ASME/ASEE 26th Joint Propulsion Conference, Orlando, FL, July 16-18, 1990; received July 30, 1990; revision received Feb. 14, 1991; accepted for publication Feb. 15, 1991. Copyright © 1990 by the American Institute of Aeronautics and Astronautics, Inc. All rights are reserved.

^{*}Associate Professor of Mechanical Engineering. Member AIAA. †Staff Engineer, Aircraft Engines. Member AIAA.

[‡]Engineer, Aircraft Engines.

malized distance (x/s) downstream of slot breakout (x = 0)are key parameters to which film effectiveness (η_i) is correlated. Other parameters, however, such as slot to mainstream flow density ratio (ρ_c/ρ_h) , slot lip thickness to height ratio (t/s), slot width to height ratio (w/s), slot angle of injection relative to mainstream flow (α) , and mainstream flow acceleration are recognized as having effects of various magnitudes on slot film effectiveness (η_t) but are not as well established. Wieghardt² conducted an experimental study injecting heated flow out of slots angled 30 deg to the mainstream. For small temperature differences between mainstream and slot flow. as was the case in Wieghardt's study, where the flow can be considered of constant properties, the film effectiveness should be independent of whether the slot flow is hotter or colder than the mainstream flow. Hartnett³ tested the same geometry as Wieghardt's while injecting a cooling flow with similarly small temperature differences and obtained similar results. Seban et al.4 tested tangential slot heating flow with a very small slot lip thickness to height ratio and small blowing ratios.5 Samuel and Joubert6 measured the film effectiveness on a flat plate in zero pressure gradient downstream of a tangential injection slot. Cold secondary flow was injected into the hot mainstream through a slot with a knife-edge-shaped lip. Their results are compared with those obtained in the present investigation. When there is a large temperature difference between mainstream and slot flow, where constant property flow cannot be assumed, it is not apparent that the film effectiveness will be independent of whether the slot flow is hotter or colder than the mainstream flow. Goldstein suggested that density ratio between mainstream and slot flow is a key parameter in understanding film effectiveness. Papell and Trout⁷ tested with tangential slot cooling flow for a number of density ratios (large and small) with a very small lip thickness to height ratio. Large density ratio tests such as this generally require metal test rigs (because of high mainstream temperatures) which make the attainment of adiabatic wall temperatures very difficult due to high metal thermal conductivity. Kacker and Whitelaw8,9 investigated the influence of slot height, slot-lip-thickness and slot turbulence intensity on the film effectiveness for a uniform density, two-dimensional wall jet. Papell¹⁰ also investigated the effects of slot angle of injection on film effectiveness and made measurements for $\alpha = 45$, 80, and 90 deg. The measurements were carried out with large density ratios, small t/s ratios, and Mach numbers mainly greater than 0.5. Sivasegaram and Whitelaw¹¹ measured film cooling effectiveness for a similar slot configuration but with $\alpha = 30, 60,$ and 90 deg. Both investigations indicated a significant decrease in η_f as α increased. Goldstein et al. 12 conducted an experimental study of film cooling effects produced by injection of helium and air secondary flows through a porous section into a turbulent boundary layer of air flowing over a flat plate. Nicoll and Whitelaw13 measured the effectiveness of a two-dimensional wall jet by injecting helium through a slot into an air mainstream for a range of slot to freestream mass velocity ratios. A comparison was also made between the experimental data and their predicted results. Burns and Stollery¹⁴ investigated the influence of foreign gas injection and slot geometry on film cooling effectiveness. They measured the foreign gas concentration for a range of velocity and density ratios. Rastogi and Whitelaw¹⁵ and Patankar et al. 16 reported experimental and numerical results for the adiabatic wall effectiveness downstream of three-dimensional slots made up of discrete holes discharging parallel to the mainstream. Paxson and Mayle17 performed theoretical and experimental investigations on the influence of the mainstream thermal boundary layer on film effectiveness.

A number of investigators have reported on the measurement of heat transfer coefficient in the region immediately downstream of slot breakout. Metzger et al. 18 reported the results of an experimental investigation on heat transfer with film cooling near flush, angled slots. Ballal and Lefebvre¹⁹ conducted an experimental investigation in heat transfer coefficient and film effectiveness downstream of a slot in an annular geometry pertinent to film-cooled combustors. Pai and Whitelaw²⁰ developed a procedure for the prediction of adiabatic wall temperature and heat transfer coefficient downstream of two-dimensional film cooling slots and compared their predicted results with the experimental data obtained by a number of investigators.

Most experimental studies on slot film cooling have been investigations with a single slot geometry and for large x/s values where flow has reached a fully developed turbulent boundary layer profile. The emphasis in this investigation was to study the effect different slot geometries have on η_f in the vicinity of slot breakout region where the flow structure is quite complex. It is in this region that slot η_f results will be directly applicable for turbine airfoil cooling design.

Experimental Apparatus and Procedure

A detailed drawing of the experimental test rig is shown in Fig. 2. The test rig consisted of a mainstream air supply, a slot air supply, a multi-slot test section, and associated measuring instruments. A compressor station supplied the mainstream and slot air to the test rig. The compressed mainstream air was filtered, dried, and directed through a heater where its temperature was raised to a constant level of about 250°F. The air was then directed through a flow measuring station, fabricated in accordance with ASME specifications, and into the test rig mainstream plenum chamber through a diffuser section. Similarly, the slot or film cooling air after being filtered and dried was routed through flow control valves, a flow measuring station, and into the test rig slot plenum chamber. Because of its low thermal conductivity, the test rig was fabricated out of wood (maple) in order to minimize the thermal conduction effects.

The slot air was fed from the slot plenum into nine parallel channels that formed the slots upon breakout into the mainstream flow.

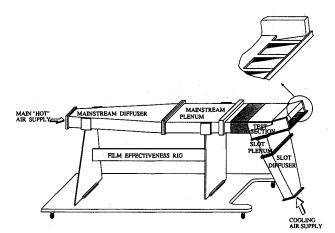


Fig. 2 Detailed drawing of test rig (not to scale).

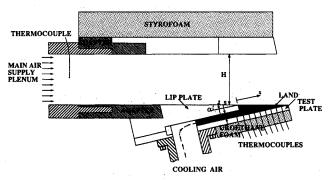


Fig. 3 Cross-sectional view of test section for angled film cooling injection ($\alpha=8.5$ deg) with accelerating mainstream flow.

Test section	α, deg	t, in.	<i>H</i> , in.	t/s	w/s	X_{max}/s	М	T_h/T_c
1	0.00	0.125-0.3125	2.93	0.5-1.25	2-17	11.00	0.4-1.6	1.20-1.60
2	0.00	0.250	2.93	1.0	5.00	16.00	0.2 - 1.4	1.24 - 1.31
3	5.00	0.175	2.98	1.0	7.14	22.86	0.2 - 1.4	1.22 - 1.26
4	8.50	0.299	2.90	1.0	4.18	13.38	0.2 - 1.4	1.25 - 1.30
5	11.5	0.407	2.90	1.0	3.07	9.830	0.2 - 1.4	1.24 - 1.29
6	15.0	0.536	2.93	1.0	2.33	7.460	0.2-1.4	1.28-1.33

Table 1 Specifications of different test sections

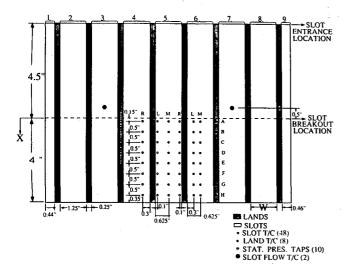


Fig. 4a Test plate instrumentation.

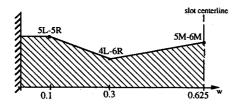


Fig. 4b Weighted average of η_f across the slot width.

In order to test 11 different slot geometries, six different test sections were used. The test sections were designed as modules that allowed simple assembly (gaskets and bolts) into the test rig. A cross section of the test section is shown in Fig. 3. Test section 1 was used to test different density ratios (ρ_c/ρ_h) and different slot geometries (t/s and w/s) with a nonaccelerating mainstream flow parallel to the slot flow (α = 0). For this test section, 4 different slot lip thickness to height ratios (t/s = 0.5, 0.75, 1.0, and 1.25), 3 different slot width to height ratios (w/s = 2, 5, and 17) and 5 different density ratios ($\rho_c/\rho_h = 1.2, 1.3, 1.4, 1.5, \text{ and } 1.6$) were tested. Test sections 2 through 5 were used to test 5 different coolant to mainstream flow angles ($\alpha = 0, 5, 8.5, 11.5,$ and 15 deg). Test sections 2 through 6 were all tested at constant values of ρ_c/ρ_h and t/s of 1.3 and 1.0, respectively. Table 1 shows the specifications of test sections 1 through 6. The slot lip plate, which separates the coolant from the mainstream flow upstream of the slot breakout location, rests on eight wooden lands. Eight grooves were made into the 1-in.-thick test plate to allow easy installation and removal of the lands as well as the lip plate which allowed different t/s and w/s configurations to be tested. For test section 1, where $\alpha = 0$ deg, the slot height (s) was held constant at 0.25 in. and t/s was varied by using different thickness lip plates.

Two-inch-thick Styrofoam insulation sheets were placed on the top and side walls of the test section and a 2-in.-thick layer of urethane foam was sprayed under the test plate to minimize heat losses to the surroundings. For the angled tests, four new lip plates, sets of lands, and adapter sections were manufactured corresponding to the four angles of injection ($\alpha = 5.0, 8.5, 11.5, \text{ and } 15.0 \text{ deg}$). For these tests, in order to maintain the slot lip thickness to height ratio constant (t/s = 1), the slot height varied as a function of the injection angle (α).

Test sections were instrumented identically. The locations of thermocouples are shown schematically in Fig. 4. Six columns of eight type K thermocouples were located 0.5 in. apart with the first thermocouple located 0.15 in. from the slot breakout. These surface temperatures were used to measure adiabatic wall temperatures (T_{aw}) . The temperature of the slot coolant flow (T_c) was measured with thermocouples immersed 0.5 in. upstream of the slot breakout on the axial centerline of slots 3 and 7. The mainstream air temperature (T_h) was measured with three thermocouples immersed evenly across the width of the inlet section.

For each test configuration, a test run was made by holding all parameters constant throughout the run except the blowing ratio (M).

A typical test was performed as follows: the mainstream temperature was set to 250°F (except for the density ratio test) and the mainstream mass flow rate at 1 lbm/s. For the test with the converging nozzle the mainstream flow rate was set to 1.5 lbm/s. The slot flow rate was set such that the blowing ratio was approximately 0.2. For each blowing ratio, sufficient time was allowed for the system to reach equilibrium at which time temperature and pressure measurements were recorded. The slot cooling flow was then increased for a larger blowing ratio, and data was recorded again. This process was repeated until measurements were obtained for a range of blowing ratios from 0.2 to 1.4. The slot Reynolds number, Re_s, varied from 1400 to 12,000 depending on the blowing ratio. The process was then reversed, and measurements were obtained for decreasing blowing ratios to investigate any hysteresis effects related to the test rig. Thermocouple and pressure transducer outputs were recorded using a Motorolla 6800 high-speed digital data acquisition system directly connected to a Honeywell H6000 mainframe. In this manner data was acquired and reduced immediately. Film effectiveness measurements were calculated directly from the experimental data

$$\eta_f = \frac{T_h - T_{aw}}{T_h - T_c} \tag{1}$$

Even with a well-insulated low thermal conductivity material (wood), some heat will be lost from the test section due to conduction and radiation. Hence, some error in η_f is introduced with the assumption that the measured surface temperatures are the idealized adiabatic wall temperatures. Of those losses, conduction through the test section surface and urethane insulation was the larger. The error due to conduction is proportional to the difference between the ambient and surface temperatures and is small (calculated maximum of 2%). The reported results for η_f are the slot surface area weighted averages and are calculated from individually measured η_f values consistent with the designations in Figs. 4a and 4b as follows:

$$\eta = [0.1\eta_{5L,5R} + 0.2(\eta_{5L-5R} + \eta_{4R-6L})/2
+ 0.325(\eta_{4R-6L} + \eta_{5M-6M})/2]/0.625$$
(2)

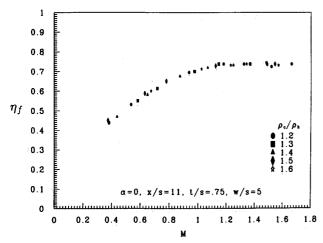


Fig. 5 Film effectiveness (η_f) vs blowing ratio (M) for different ρ_c/ρ_h ratios.

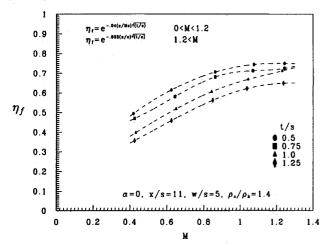


Fig. 6 Film effectiveness (η_f) vs blowing ratio (M) for different t/s ratios.

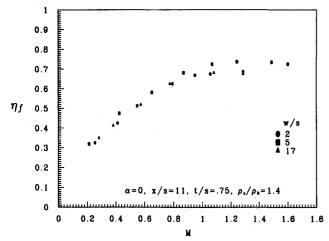


Fig. 7 Film effectiveness (η_f) vs blowing ratio (M) for different w/s

Experimental uncertainties for the film effectiveness and the mass-velocity ratio, following the method of Kline and McClintock,²¹ were found to be $\pm 4\%$ and $\pm 6.5\%$, respectively.

Results and Discussion

Density Ratio

For this investigation, the wooden test apparatus dictated an upper limit for the mainstream gas temperature, beyond

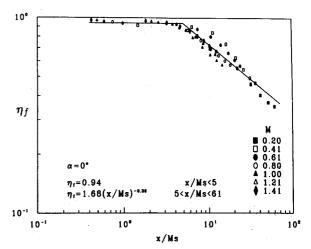


Fig. 8 Film effectiveness (η_f) vs blowing ratio (M) for 0 deg injection angle.

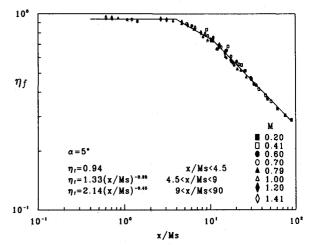


Fig. 9 Film effectiveness (η_f) vs blowing ratio (M) for 5 deg injection angle.

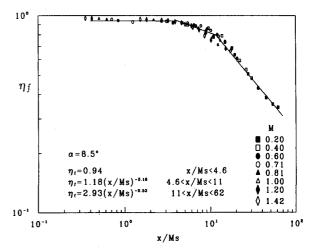


Fig. 10 Film effectiveness (η_f) vs blowing ratio (M) for 8.5 deg injection angle.

which the rig would be seriously damaged. Past investigations have suggested that temperature and density ratios between mainstream and coolant flow may have a significant influence on the behavior of slot film effectiveness. This, along with the fact that cooling film from slots in modern turbine airfoils exits within temperature ratios from 1.2 to 1.6, required the necessity to determine the effect density ratio had on η_f . Figure 5 shows that film effectiveness is insensitive to density ratio (ρ_c/ρ_h) in the tested range of 1.2 to 1.6. All geometric parameters were held constant for these tests. The reported

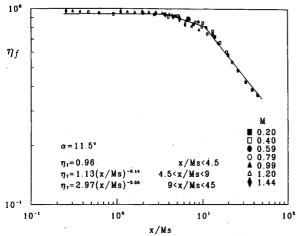


Fig. 11 Film effectiveness (η_f) vs blowing ratio (M) for 11.5 deg injection angle.

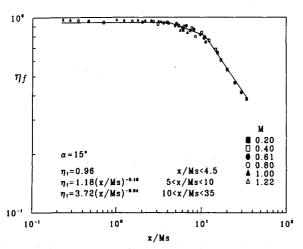


Fig. 12 Film effectiveness (η_f) vs blowing ratio (M) for 15 deg injection angle.

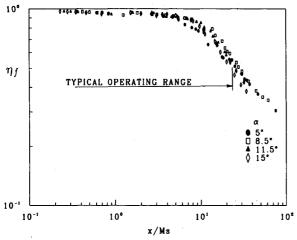


Fig. 13 Film effectiveness (η_f) vs (x/Ms) for varying injection angles $(\alpha = 5, 8.5, 11.5, 15 \text{ deg})$.

x/s location in Fig. 5 is 11 and was selected arbitarily from other measured x/s locations since the effect of (ρ_c/ρ_h) on η_f for each x/s location was similar. Based on these results, the remainder of tests in this investigation were performed at a near constant density ratio (1.4) where the mainstream gas temperature was held at approximately 250°F.

Lip Thickness to Slot Height Ratio

The thickness of the slot lip separating the mainstream and coolant flows has been shown in previous investigations to

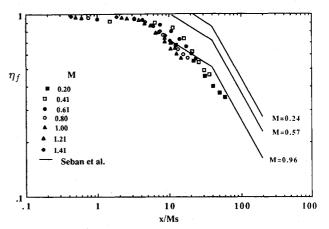


Fig. 14 Comparison between the present data and those obtained by Seban et al.⁴

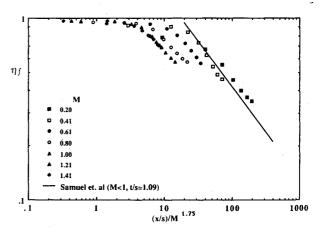


Fig. 15 Comparison between the present data and those obtained by Samuel and Joubert.⁶

have an effect on η_f . Most relative studies have been for very small lip thickness to slot height ratios (t/s) which are well below the values that exist in modern turbine airfoils, where realistic t/s ratios of 0.5 to 1.25 occur. To determine the effect t/s has on η_f , four different geometries were tested, where all parameters except lip thickness (t) were held constant. As expected, Fig. 6 shows that as the lip thickness increases, film effectiveness decreases. It is believed that for very small lip thicknesses (i.e., knife-point), mixing between the mainstream and coolant flow will be minimized and η_f will be maximized. As the lip thickness gets larger, eddy mixing between the mainstream and coolant flow increases, resulting in lower η_f . The results in Fig. 6 show that the effect of t/s on η_f can be reasonably correlated with t/s as an exponential square root function as follows:

$$\eta_f = e^{-0.04(x/Ms)\sqrt{t/s}} \tag{3}$$

for $0 < M \le 1.2$

$$\eta_f = e^{-0.033(x/Ms)\sqrt{t/s}} \tag{4}$$

for $1.2 \le M$

Slot Width to Height Ratio

As is usually the case for turbine airfoil trailing edge slot film cooling, the slot geometry and number of slots is dictated by airfoil size and the amount of allowable cooling flow. Large slot widths relative to slot heights (resulting in large w/s ratios) are mechanically unfeasible. Conversely, small slot width to height ratios may induce nonuniform radial cooling. Furthermore, slot endwall effects may have a significant influence on η_f as w/s decreases. To determine the effect w/s has on

 η_f , three different w/s geometries were tested, where all parameters except slot width (w) were held constant. Figure 7 shows that for w/s in the range of 2–17, as is typically found in modern turbine airfoils, η_f is basically insensitive to w/s.

Injection Angle

The angle at which the coolant flow is injected (α) relative to the mainstream is another geometric parameter that is largely dictated by airfoil size and shape. Previous investigations have been made for various angles but not on a comparative basis in the typical range found in modern turbine airfoils. To determine the effect α has on η_t , five different α 's in the range 0-15 deg were tested. For all five tests, ρ_c/ρ_h , t/s and w/s were held constant. Figures 8 through 12 show the results of each individual α where η_c is plotted versus x/Ms on a log-log basis. x/Ms was selected to plot against, as it has been found in the past to be a convenient dimensionless parameter that tends to collapse η_t data into a single curve. For each α , a set of correlations are offered in order that quick calculations can be made to estimate η_f based on the data. For these correlations, the constant and the exponent of the power law relation were evaluated by performing a least-squares fit to the data for a range of x/Ms values. A plot of η_f as a function of x/Ms is presented in Fig. 13 for different injection angles. As can be seen from this figure, 8.5 deg appears to be an optimum angle (although slight) for typical operating values of x/Ms.

Comparisons with Previous Work

Experimental results obtained for the tangentially injected flow configuration ($\alpha=0$) are compared with those obtained by Seban et al.⁴ in Fig. 14. Data obtained in this study are lower than those reported by Seban et al. The difference is due to different slot geometries. A very small lip thickness in the Seban et al. experiment corresponds to a very small t/s ratio while this ratio in the present study was unity and its strong effect on film effectiveness has been established (Fig. 6). Figure 15 shows a comparison between the present results and those of Samuel and Joubert.⁶ Excellent agreement is found for small M ratios bearing in mind that Samuel and Joubert's correlation pertains to greater downstream distances than the present results.

Summary and Conclusions

A parametric investigation was performed to experimentally determine what effect the following parameters have on airfoil slot film effectiveness: coolant to mainstream density ratio (ρ_c/ρ_h) ; slot lip thickness to height ratio (t/s); slot width to height ratio (w/s) and coolant to mainstream angle of injection (α) . The results indicate the following: 1) For the range of density ratios tested (1.2 $\leq \rho_c/\rho_h \leq$ 1.6), η_f is insensitive to the density ratio; 2) For the range of lip thickness to slot height ratios tested (0.5 $\leq t/s \leq$ 1.25), η_t is strongly sensitive to t/s and can be correlated to an exponential square root function of t/s given by Eqs. (3) and (4); 3) For the range of slot width to height ratios tested ($2 \le w/s \le 17$), η_f is insensitive to w/s; and 4) For the range of coolant to mainstream injection angles tested (0 deg $\leq \alpha \leq$ 15 deg), η_t is slightly sensitive to α where 8.5 deg appears to be the optimum angle.

Acknowledgments

The authors wish to express gratitude to R. E. Gladden, J. D. Cohen, and H. L. Morton for their support, expertise, and encouragement during this investigation.

References

¹Goldstein, R. J., "Film Cooling," Advances in Heat Transfer, Academic Press, New York and London, Vol. 7, 1971, pp. 321–379. ²Wieghardt, K., "Hot-Air Discharge for De-Icing," A.A.F. Translation F-TS-919-RE, 1946.

³Hartnett, J. P., Birkebak, R. C., and Eckert, E. R. G., "Velocity Distributions, Temperature Distributions, Effectiveness and Heat Transfer for Air injected Through a Tangential Slot Into a Turbulent Boundary Layer," *American Society of Mechanical Engineers Journal of Heat Transfer*, Series C, Vol. 83, No. 3, 1961, pp. 293–306.

⁴Seban, R. A., Chan, H. W., and Scesa, S., "Heat Transfer to a Turbulent Boundary Layer Downstream of an Injection Slot," Paper 57-A-36 American Society of Mechanical Engineers Annual Meeting, 1957.

⁵Seban, R. A., "Heat Transfer and Effectiveness for a Turbulent Boundary Layer with Tangential Fluid Injection," *American Society of Mechanical Engineers Journal of Heat Transfer*, Series C, Vol. 82, No. 4, 1960, pp. 303-312.

⁶Samuel, A. E., and Joubert, P. N., "Film Cooling of an Adiabatic Flat Plate in Zero Pressure Gradient in the Presence of a Hot Mainstream and Cold Tangential Secondary Injection," American Society of Mechanical Engineers Paper 64-WA-HT-48, 1964.

of Mechanical Engineers Paper 64-WA-HT-48, 1964.

Papell, S. S., and Trout, A. M., "Experimental Investigation of Air Film-Cooling Applied to an Adiabatic Wall by Means of an Axially Discharging Slot," NASA TN-D-9, 1959.

*Kacker, S. C., and Whitelaw, J. H., "The Effect of Slot Height

*Kacker, S. C., and Whitelaw, J. H., "The Effect of Slot Height and Slot Turbulence Intensity on the Effectiveness of the Uniform Density, Two Dimensional Wall Jet," American Society of Mechanical Engineers Journal of Heat Transfer, 1968, pp. 469-475.

⁹Kacker, S. C., and Whitelaw, J. H., "An Experimental Investigation of the Influence of Slot-Lip-Thickness on the Impervious-Wall Effectiveness of the Uniform-Density, Two-Dimensional Wall Jet," *International Journal of Heat and Mass Transfer*, Vol. 12, 1969, pp. 1201–1206.

 ¹⁰Papell, S. S., "Effect on Gaseous Film Cooling Injection Through Angled Slots and Normal Holes," NASA TN-D-299, 1960.
 ¹¹Sivasegaram, S., and Whitelaw, J. H., "Film Cooling Slots: the

¹¹Sivasegaram, S., and Whitelaw, J. H., "Film Cooling Slots: the Importance of Lip Thickness and Injection Angle," *Journal of Mechanical Engineering Science*, Vol. 11, No. 1, 1969.

¹²Goldstein, R. J., Rask, R. B., and Eckert, E. R. G., "Film

¹²Goldstein, R. J., Rask, R. B., and Eckert, E. R. G., "Film Cooling with Helium Injection into an Incompressible Air Flow," *International Journal of Heat and Mass Transfer*, Vol. 9, 1966, pp. 1341–1350.

¹³Nicoll, W. B., and Whitelaw, J. H., "The Effectiveness of the Uniform Density, Two-Dimensional Wall Jet," *International Journal of Heat and Mass Transfer*, Vol. 10, 1967, pp. 623-639.

¹⁴Burns, W. K., and Stollery, J. L., "The Influence of Foreign Gas Injection and Slot Geometry on Film Cooling Effectiveness," *International Journal of Heat and Mass Transfer*, Vol. 12, 1969, pp. 935–951

¹⁵Rastogi, A. K., and Whitelaw, J. H., 1973, "The Effectiveness of Three-Dimensional Film-Cooling Slots-I. Measurements," *International Journal of Heat and Mass Transfer*, Vol. 16, 1973, pp. 1665–1672

¹⁶Patankar, S. V., Rastogi, A. K., and Whitelaw, J. H., "The Effectiveness of Three-Dimensional Film-Cooling Slots-II. Predictions," *International Journal of Heat and Mass Transfer*, Vol. 16, 1973, pp. 1673–1681.

¹⁷Paxson, D. E., and Mayle, R. E., "The Influence of a Mainstream Thermal Boundary Layer on Film Cooling Effectiveness," ASME Paper 88-GT-17, 1988.

¹⁸Metzger, D. E., Carper, H. J., and Swank, L. R., "Heat Transfer with Film Cooling Near Nontangential Injection Slots," *American Society of Mechanical Engineers Journal of Engineering for Power*, 1968, pp. 157–163.

¹⁹Ballal, D. R., and Lefebvre, A. H., "A Proposed Method for Calculating Film-Cooled Wall Temperature in Gas Turbine Combustion Chambers," American Society of Mechanical Engineers Paper 72-WA/HT-24, 1972.

²⁰Pai, B. R., and Whitelaw, J. H., "The Prediction of Wall Temperature in the Presence of Film Cooling," *International Journal of Heat and Mass Transfer*, Vol. 14, 1971, pp. 409-426.

Heat and Mass Transfer, Vol. 14, 1971, pp. 409-426.

²¹Kline, S. J., and McClintock, F. A., "Describing Uncertainties in Single-Sample Experiments," Mechanical Engineering, Vol. 75, No. 1, 1953, pp. 3-8.